Abstract: In this paper, an aggregate production-planning problem is formulated as a stochastic linear quadratic problem with chance constraints on state and control variables. The stochastic model proposed extends the classical aggregate model developed by Holt, Modigliani, Muth and Simon (HMMS). As an example of application, an equivalent deterministic problem is developed from the stochastic model. This deterministic problem provides an open-loop solution that allows managers to get important insight about the use of the company’s material resources.

Keywords: Production planning, make-to-stock system, decision-making, stochastic control, quadratic programming.

1. INTRODUCTION

A production planning problem requires a set of decisions that is used to adapt the company’s industrial resources in order to satisfy the demand of its products. The decisions are made over different time scales (i.e., long, medium, and short-term horizon) that are associated with, at least, three levels of the hierarchical planning process (i.e., strategic, tactical and operational levels, see (Hax and Candea, 1984)). For instance, at the higher planning level (strategic level, therefore), many decisions are usually taken over a long-term horizon (i.e. from 1 to 2 years). In such level, the production can only be elaborated in aggregate patterns (Bensoussan et al, 1983).

The linear decision rule developed by Holt, Modigliani, Muth and Simon – HMMS – (1960) can be considered as an important contribution for long-term strategic production planning decisions. This analytical rule is determined from the minimization of quadratic cost functions subject to inventory and workforce balance equations. As a result, it provides an optimal smoothing solution for aggregate inventory, production and workforce levels.

The HMMS model has been extensively used in the literature (see Singhal and Singhal (1996)). It is usually applied as a benchmarking tool for comparing different aggregate production planning approaches, and, as well to provide managers with insights about the use of the company’s material resources. However, there are doubts about its applicability for practical industrial purposes (APICS, 1994): one considers that quadratic cost functions are not a valid assumption, and another points out that it fails to provide a reliable production planning since it does not take into account constraints on the decision variables. The first criticism is overthrown by the argumentation that quadratic costs are interesting way for evaluating the production process (Hax and Candea, 1984), for example: quadratic inventory costs (i.e. holding cost) are incurred for both negative (backorder of sales) and positive inventory (Parlar, 1984). The second criticism, in our opinion, is essentially more realistic than the previous. In fact, for practical production planning application, does not include physical constraints into the problem can lead to solutions that can be a disaster for the company managerial purposes.

In this paper, it is shown that it is possible to obtain a sequential optimal solution for the HMMS model under constraints on decision variables. For this purpose, the original quadratic model is converted into a space-state format as described by Shen (1994) but, however, having inventory, production and workforce variable values taken from physical sets. In particular, due to the random fluctuation of demand over long-term horizons, the constraints on inventory and workforce are taken into probability in order to guarantee their feasibility. Furthermore, assuming the inventory balance system as being a Gaussian process, the resulting stochastic problem can be described as a Linear Quadratic Gaussian problem with constraints.

Because of structural features like constraints and stochastic nature of the system, this problem can become hard to be solved directly by techniques from stochastic control theory. Thus, for practical application, can be required to use certain approximate schemes that enable the model to provide a quasi-optimal solution. In this case, assumptions like the linear-Gaussian nature of the system and the quadratic criterion make possible the use of the certainty-equivalence principle (Bertesekas, 1995) which allows the transformation of the stochastic problem into a deterministic equivalent whose all decision variables are set equal to their average values. This deterministic problem is easier to be solved than the stochastic problem and the main
original properties, like linearity and convexity, are preserved. A simple example, based on HMMS data, illustrates the model’s applicability.

2. MODELING THE PRODUCTION PROBLEM

In this section, the state-space stochastic optimal control problem with constraints is formulated. It is used to represent an aggregated production planning problem with constraints on inventory, production and work force variables. This optimization model is developed from the classical HMMS model.

2.1. Notation

The HMMS model deals with aggregate patterns of different products that are grouped into single families based on similarities as, for example: shape, rates of different products that are grouped into single families. For each family the decision variables, constraints and cost functions are defined as follows:

- Decision variables:
  - \( I_k \) ending inventory at period \( k \);
  - \( P_k \) production rate at period \( k \); and
  - \( W_k \) work-force size at period \( k \).

- Demand variable:
  - \( D_k \) denotes sales level at period \( k \), that, for long-term horizon (without lost of generality) can be assumed as a stationary Gaussian random variable, with mean \( \bar{D}_k \) and variance \( \sigma^2_k \).

- Cost Components (Hax and Candea, 1984):
  - \( C_1 \cdot W_k + C_{13} \) regular payroll cost;
  - \( C_2 \cdot (W_k - W_{k-1} - C_{11})^2 \) hiring and firing costs;
  - \( C_7 \cdot (P_k - C_4 \cdot W_k)^2 \) inventory and backorder costs;
  - \( C_5 \cdot (P_k - C_6 \cdot W_k)^2 + C_3 \cdot P_k - C_6 \cdot W_k + C_2 \cdot P_k \cdot W_k \) overtime and idle costs.

Note that the coefficients \( C_i \) denote prices and their estimate values require time-consuming activities like statistical analysis, accounting information, and managerial insights.

- Constraints:
  - \( \bar{I}_k \) and \( \bar{I}_k \) lower and upper boundaries of inventory ;
  - \( \bar{P}_k \) and \( \bar{P}_k \) lower and upper boundaries of production ; and
  - \( \bar{W}_k \) upper boundary of workforce .

It is important to observe that the above lower and upper bounds are not considered in the original HMMS model. Thus, in order to overcome such deficiency, physical constraints related to inventory, production and workforce variable are introduced in this study.

2.2. The problem formulation

From the above notations, the aggregate stochastic linear-quadratic problem with probabilistic constraints can be formulated as follows:

**Functional Cost:** the HMMS cost is given by:

\[
J_k(I_k, P_k, W_k) = (C_1 - C_6) \cdot W_k + C_2 \cdot (W_k - W_{k-1})^2 + C_5 \cdot (P_k - C_4 \cdot W_k)^2 + C_3 \cdot P_k + C_12 \cdot P_k \cdot W_k + C_7 \cdot (I_k - C_8)^2 \tag{1}
\]

After some algebraic handling, it is possible to write (1) into an equivalent matricial format:

\[
J_k() = \frac{1}{2} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} I_k - \theta_1 & P_k - \gamma_1 \end{bmatrix} \begin{bmatrix} h_{11} & 0 \\ 0 & h_{22} \end{bmatrix} \begin{bmatrix} I_k - \theta_1 & P_k - \gamma_1 \end{bmatrix} + \begin{bmatrix} W_{k-1} - \theta_2 & W_{k-1} - \theta_2 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & e \end{bmatrix} \begin{bmatrix} W_{k-1} - \theta_2 & W_{k-1} - \theta_2 \end{bmatrix}
\]

where the matrices coefficients are determined, basically, comparing the expressions (1) and (2). These coefficients are given by:

- \( h_{11} = 2 \cdot C_7 \) and \( h_{22} = 2 \cdot C_5 \), \( e = -2 \cdot C_2 \),
- \( r_{11} = 2 \cdot C_{13} \), \( r_{12} = r_{11} = C_2 \cdot C_4 \) and \( r_2 = 2 \cdot (C_2 + C_3 \cdot C_4) \),
- \( \theta_i = C_s \) and \( \theta_j = \frac{\theta_i \cdot (C_s - C_i) + r_{1i}}{h_{1i} \cdot r_{11} \cdot r_{12} + r_{12} \cdot r_{11}} \),
- \( \gamma_i = -\frac{1}{r_{1i}} \cdot (C_s + r_{1i} \cdot \theta_i) \) and \( \gamma_j = \theta_j \).

It is worth observing that the transformation of HMMS costs into matrices is not as immediate as it can be seen at first glance. The main reason is due to the inconsistencies in determining some of the matrices’ coefficients which are responsible for preserving the proportionality between the matricial cost format (2) and HMMS cost (1) (Shen, 1994). In order to reach such proportionality, the equality \( C_{12} = C_1 \cdot C_4 \) must be considered. Note that such artifice does not take out the originality of the HMMS costs. In fact, the original value of the second term of the equality is relatively close to the value of \( C_{12} \) (Holt et al., 1960).

Defining now \( x_{ik} = I_k \) and \( x_{2k} = W_k \) as state variables and \( u_{ik} = P_k \) and \( u_{2k} = W_k \) as control variables, the space state representation for the system can be provided as follows:
The Balance Equation: the system that represents the inventory and workforce balance equations is given as follows:

\[
\begin{bmatrix}
    x_{1,k+1} \\
    x_{2,k+1}
\end{bmatrix}
= \begin{bmatrix}
    1 & 0 \\
    0 & 0
\end{bmatrix}
\begin{bmatrix}
    x_{1,k} \\
    x_{2,k}
\end{bmatrix}
+ \begin{bmatrix}
    1 & 0 \\
    0 & 1
\end{bmatrix}
\begin{bmatrix}
    u_{1,k} \\
    u_{2,k}
\end{bmatrix}
+ \begin{bmatrix}
    -d_{k} \\
    v_{k}
\end{bmatrix}
\]

where the first row of (3) denotes the inventory balance equation, that is, \( I_{k+1} = I_{k} + P_{k} - D_{k} \). Note also that \( d_{k} = D_{ik} \) denotes the uncertainty about the fluctuation of demand.

The second row describes the behavior of the workforce, that is, \( x_{2,k+1} = W_{k} = u_{2,k} \). The variable \( v_{k} \) contains the random variation related to workforce variable that is assumed normally distributed with mean 0 and variance \( \sigma^{2} \). The fluctuation of demand \( d_{k} \) and the workforce noise \( v_{k} \) bring randomness to the balance equation (3). As an immediate result, both inventory \( x_{1k} \) and workforce \( x_{2k} \) variables are random variables with probabilistic distribution functions dependent on the nature of the stochastic process (3). Assuming this process as being Gaussian, the distribution of variables \( x_{1k} \) and \( x_{2k} \) will be completely identified by computing from (3) the first and the second statistical moments, that is, respectively the mean and variance – \((\tilde{x}_{1k}, \tilde{x}_{2k})\) and \( (V_{x_{1}}, V_{x_{2}})\) – evolving over the periods of planning horizon.

The Constraints: It is assumed that the inventory level \( x_{1k} \), the production rate \( u_{1k} \), and the workforce level \( x_{2k} \) must keep their values within specific bounds. Since equation (3) is an stochastic process, becomes impossible to guarantee that the inventory and production variables will not be violate their constraints in any period of planning horizon. To overcome such difficulty, a chance-constraints on inventory and production variables are considered. Follows then that the set of constraints employed in this formulation are:

\[
\begin{align*}
\text{Prob}(\bar{x}_{n} \leq x_{n} \leq \bar{x}_{n}) & \geq \alpha \\
\text{Prob}(\bar{x}_{n} < x_{n}) & \geq \beta \\
\text{Prob}(u_{n} \leq u_{n} \leq \bar{u}_{n}) & \geq \eta \text{ and } u_{n} \geq 0
\end{align*}
\]

where \( \bar{x}_{nk} = \tilde{I}_{k} \) denotes the safety stock levels; \( \bar{x}_{nk} = \bar{I}_{k} \) denotes the maximal storage capacity allowed over the periods; \( u_{nk} = P_{k} \) and \( \bar{u}_{nk} = \bar{P}_{k} \) denote the minimal and maximal production capacity; and \( \bar{x}_{nk} = \bar{W}_{k} \) denotes the maximal level of workforce allowed by the company’s managers. The indexes \( \alpha, \beta \) and \( \eta \) are measures of probability chosen by the manager in the range \((0, 1]\). The parameter \( \alpha \) can be interpreted as the tradeoff between backlogged sales (by choosing \( \alpha \in [0,1/2] \) and customer satisfactions (\( \alpha \in [1/2,1] \)). For example, if the manager chooses \( \alpha = 0.9 \) means that he expects to satisfy demand at least 90% of time (i.e., high customer service level). Otherwise, choosing \( \alpha = 0.1 \) means the possibility of backlogging occurrence at least 90% of time (i.e., low customer service level). As a consequence, by varying \( \alpha \) in the range \([1/2,1] \), the manager can study different policy of improve customer service (Silva Filho, 1999).

Similarly, the parameter \( \beta \) indicates that the probability of the workforce level’s fluctuation does not overcome a desired pre-specified level of labor. Both parameter \( \alpha \) and \( \beta \) are previously defined by the manager and they can help to define production long-term goals to be reached by the company.

Concerning with constraints (4) yet, it is observed that the production variable \( u_{1k} \) is also taken in probability. This occurs because of production level takes into account, at each period of the planning horizon, the actual position of inventory level \( x_{1k} \) (i.e. \( u_{1k} = \mu_{k}(x_{1k}) \), where \( \mu \) denotes a function that relates \( x_{1k} \) with \( u_{1k} \)).

Note that if it is assumed that such dependence is linear (a linear closed-loop scheme), then the control variable \( u_{1k} \) will be seen as random variable whose distribution follows a similar distribution pattern of the state variable \( x_{1k} \) (i.e., a Gaussian distribution). In this case, the control variable will be characterized by a mean \( \bar{u}_{nk} \) and variance \( V_{uk} \geq 0 \).

Therefore, in order to guarantee the feasibility of optimal solution, the constraint of variable \( u_{1k} \) is taken in probability and the parameter \( \eta \) can be interpreted as a degree of utilization of the production capacity (for example: \( \eta = 0.9 \) means that aggregate capacity of the process can operates with the maximal load available at least 90% of time).

2.3. Optimal Solution

Now the stochastic optimal control problem with constraints, to be applied to an aggregate production planning problem, can be formulated. Basically, the objective is to determine the closed-loop optimal control sequence \( (u_{1k}^{*} = \mu_{k}(x_{1k})|_{k=0,...,T-1}) \) and the open-loop sequence \( (u_{1,0}, u_{1,n}, ..., u_{1,T-1}) \) that simultaneously minimize the average functional criterion (2) subject to the stochastic linear balance equation (3) and the chance constraints (4).

Due to the stochastic nature, the constraints on decision variables and the dimensionality (for practical applications), to try to obtain a true optimal solution for this problem can become a hard task. Classical stochastic dynamic programming naturally can be used, however, because of the enormous computational effort required to solve large
dimension problems, this approach is rarely applicable in practice. Because of the above difficulties, approximate solutions (sub-optimal, therefore), that require smaller computational effort, are usually preferred (Neck, 1984). Next, an approach that transforms the stochastic problem into a deterministic equivalent is briefly discussed. It provides a sub-optimal solution for the problem (2)-(4).

3. SUB-OPTIMAL SOLUTIONS

There are several sub-optimal approaches from literature that can be used to solve the stochastic problem (2)-(4). Of course, these techniques sacrifice the true optimal policy in order to provide a feasible solution that is easier to compute and implement (Neck, 1984). Some of these techniques are based on the certainty-equivalence principle (Bertesekas, 1995). This principle explores the linear-Gaussian nature of the process (3) to transform the stochastic problem into a deterministic equivalent. This deterministic problem maintains the main properties of the original problem (for instance, convexity and linearity properties); and brings, as its main advantage, the possibility of exploiting different mathematical programming methods in order to solve it (Lassere, et al. 1984).

The simplest example of an equivalent deterministic model for the problem (2)-(4) is the one where all decision variables are set equal to their mean values (i.e. essentially based on certainty-equivalence principle). For the stochastic production planning discussed in the previous section, this deterministic problem can be stated from the following facts:

- the linearity of the economic balance equation (3);
- the convexity of the HMMS’s functional costs, described by (2); and
- the demand and workforce random variations, both described as Gaussian processes.

Assuming that the probability distributions of the random variables $d_k$ and $v_k$ are known over the periods as being Gaussian distributions, and following a basic property of stochastic process which says that: the resulting linear transformation of a sequence of Gaussian vectors is also a Gaussian process, then, it is possible to conclude that the statistical behavior of the process (3) is exactly known as a Gaussian behavior. This characteristic means that the inventory and workforce distribution functions can be precisely computed. In fact, due to the Gaussian assumption, all statistic information required to identify these distributions are determined by their mean and variance equations defined from process (3) (see Silva Filho, 1999). As a result, these statistics allow to carry out the transformation of the stochastic problem (2)-(4) into the deterministic equivalent problem.

3.1. The Transformation Process

Before setting out with this transformation, the following notations are introduced

(i) Mean variables: $E\{x_{ik}\} = \bar{x}_{ik}$, $E\{x_{ik}\} = \bar{x}_{ik}$, $u_{ik} = \hat{u}_{ik}$, $u_{ik} = \hat{u}_{ik}$, $\hat{d}_i = \hat{D}_i$; and

(ii) Variance variables: $V_{s_i} = E\{x_{ik}^2\}$, $V_{s_i} = E\{x_{ik}^2\}$, and $V_{s_i} = V_{s_i} = 0$.

Note that, from now, the control variables $u_{ik}$ (production level) and $u_{ik}$ (workforce level) are essentially deterministic. This means that they operate totally in open-loop over the periods of planning horizon.

Both quadratic criterion (2) and balance equation (3) are promptly converted to deterministic equivalent pattern (see Silva Filho, 1999). Another important transformation changes the chance constraints (4) into equivalent, but deterministic, inequalities, that is:

\[
\begin{align*}
\text{Prob}(x_{ik} \geq \bar{x}_{ik}) \geq \alpha & \Leftrightarrow \hat{x}_{ik} \geq \bar{x}_{ik} = \bar{x}_{ik} + \sqrt{V_{s_i}} \cdot \Phi_{i+1}(\alpha) \\
\text{Prob}(x_{ik} \leq \bar{x}_{ik}) \geq \alpha & \Leftrightarrow \hat{x}_{ik} \leq \bar{x}_{ik} = \bar{x}_{ik} - \sqrt{V_{s_i}} \cdot \Phi_{i+1}(\alpha)
\end{align*}
\]

where $\Phi_{i+1}(\cdot)$ denotes the inverse of the inventory variable’s probability distribution function. The analogous handling must be used to convert the probabilistic workforce constraint presented in (4), that is:

\[
\text{Prob}(x_{ik} \leq \bar{x}_{ik}) \geq \beta \Leftrightarrow \hat{x}_{ik} \leq \bar{x}_{ik} = \bar{x}_{ik} - \sqrt{V_{s_i}} \cdot \Phi_{i+1}(\beta)
\]

As a result, the deterministic optimal problem can be formulated as described in sequel by (7).

As was said before, the problem (7) provides an open-loop optimal solution. It is an approximate solution for the stochastic problem (2)-(4). Though it does not provide true optimal solution, it can be very important for managerial purposes. Indeed, it can help managers to get insights about the use of the firm’s material requirement resources. As well, it can be used as aggregate production targets to be achieved during the definition of the detailed production plan (Bensoussan et al., 1983).
\[
\text{Min } \begin{pmatrix} \hat{x}_{11} \\ \hat{x}_{21} \end{pmatrix} \begin{bmatrix} h_0 & 0 \\ 0 & h_0 \end{bmatrix} \begin{pmatrix} \hat{x}_{11} \\ \hat{x}_{21} \end{pmatrix} + 1/2 \sum_{i=1}^{7-2} \begin{pmatrix} \hat{x}_{11} \\ \hat{x}_{21} \end{pmatrix} \begin{bmatrix} h_0 & 0 \\ 0 & h_0 \end{bmatrix} \begin{pmatrix} \hat{x}_{11} \\ \hat{x}_{21} \end{pmatrix} + 2 \begin{pmatrix} \hat{x}_{11} \\ \hat{x}_{21} \end{pmatrix} \begin{bmatrix} 0 & 0 \\ 0 & e \end{bmatrix} \begin{pmatrix} \hat{u}_{11} \\ \hat{u}_{21} \end{pmatrix} + \begin{pmatrix} \hat{u}_{11} \\ \hat{u}_{21} \end{pmatrix} \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \begin{pmatrix} \hat{u}_{11} \\ \hat{u}_{21} \end{pmatrix}
\]

such that
\[
\begin{pmatrix} \hat{x}_{11,k} \\ \hat{x}_{21,k} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{x}_{11} \\ \hat{x}_{21} \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \hat{u}_{11} \\ \hat{u}_{21} \end{pmatrix} - \begin{pmatrix} \hat{d}_1 \\ \hat{d}_2 \end{pmatrix}
\]

\[
\sum_{k=1}^{n} x_{1k} \leq \bar{x}_{1k} \leq \bar{x}_{1,k} \quad \text{and} \quad \sum_{k=1}^{n} x_{2k} \leq \bar{x}_{2k} \leq \bar{x}_{2,k}
\]

\[
u_{1k} \leq \hat{u}_{1k} \leq \bar{u}_{1k} \quad \text{and} \quad \hat{u}_{2k} \geq 0
\]

\[k = 0, 1, \Lambda, T - 1\]

3.2. An Example

As an example, a multi-period, single product aggregated production planning problem is formulated by mean of model (7). The data follow partially the data provided by HMMS problem (Holt et al., 1960):

- The average demand (\( \hat{D}_k \)):

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>June</th>
</tr>
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<tbody>
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<td>431</td>
<td>444</td>
<td>442</td>
<td>317</td>
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<td>375</td>
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<td></td>
<td>392</td>
<td>500</td>
<td>492</td>
<td>450</td>
<td>395</td>
<td>415</td>
</tr>
</tbody>
</table>

- Initial/Final condition:

\[ I_0 = 100 \quad I_T = 350 \quad P_0 = 200 \quad P_T = 450 \quad W_0 = 120 \]

- The cost’s components of (1) are given by: \( C_1 = 69.7; C_2 = 64.3; C_3 = 51.2; C_4 = 13.7; C_5 = 0.0825; C_6 = 320 \), and \( C_{12} = 1.134 \). The proportionality constants are: \( \theta_1 = 320; \theta_2 = 104; \gamma_1 = 617; \text{and} \gamma_2 = 104 \).

The standard deviation of the demand is given by \( \sigma_0 = 14.14 \) and the customer satisfaction degree, associated with the inventory constraint, is set equal to 95% (\( \alpha = 0.95 \)). The variability of workforce fluctuation is defined by the standard deviation \( \sigma = 6.1 \) and the probabilistic degree to set the workforce level close to a desired upper level, related to some kind of labor policy, is assumed to be equal to 95% (\( \beta = 0.95 \)).

The deterministic optimal solution is shown in the figures 1 and 2. It is amazing to find that the HMMS model provides important insights to manage a particular type of production process known as make-to-stock system (APICS, 1994). In fact, observe in figure 1.(a) that the optimal level of inventory \( x_{1k} \) follows as much as possible its upper boundary (dashed line in figure 1.(a)). This behavior of holding high level of stock can be understood as a trial of satisfying future demand that fluctuates randomly over the periods of long-term planning horizon, and as well to of prevent future production disturbances such as: late deliveries or expired due-dates.

Besides, it is important to observe from figure 1.(b) that the workforce level does not overcome the desired level of workforce predefined by managers, that is, \( \hat{x}_{1k} \leq \bar{x}_{1k} \) where \( \bar{x}_{1k} \) is described by the dashed line. Because of the production system is operating close to the maximal capacity, the workforce levels overcome the regular labor (assumed here as RWF = 85; see the solid line in the figure 2). It is important to add that this additional quantity of workforce means the needs of adopting overtimes and/or hiring policies to keep the production operating in maximum capacity.

The results given by figures 1-2 give an brief idea of the managerial difficulties to produce a feasible production plan. The main difficulty is related to the need of satisfying tradeoffs like, for example: how to maintain an adequate
level of inventory while maximizing production rates without introducing enough overtime or hiring. Another difficulty is represented by the minimum and maximum physical boundaries of these decision variables because they mean less flexibility to modify the operating conditions whenever the production plan requires to be revised. The average optimal solution (figures 1 and 2) provided by the deterministic problem (7) can be used by the manager to obtain production planning scenarios to be considered for managerial analysis. In fact, by varying the values of $\alpha$ and $\beta$, he can extract different kinds of performance indices (like, for instance the production rates, inventory and work-force levels) that will help him in making appropriate decisions.

4. CONCLUSION

This paper described a linear-quadratic Gaussian production planning problem with chance-constraint on decision variables. This problem was built from the classical unconstrained HMMS model. Difficulties to obtain a true optimal solution led to a search for sub-optimal techniques as solution strategies. As a result of the application of the certainty-equivalence principle, an equivalent deterministic problem was introduced. Its sub-optimal solution provides important managerial insights about the use of the firm’s resources. Indeed, varying some parameters of the deterministic model (like the probabilities indexes $\alpha$ and $\beta$), the sub-optimal solution can provide different production scenarios that can help managers to establish long-term goals to be used during the development of detailed production scheduling.

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